A DYNAMIC GENERAL LINEAR MODEL FOR INFERENCE FROM ACCELERATED LIFE TESTS

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Abstract

We present a new approach for inference from accelerated life tests. Our approach is based on a dynamic general linear model setup which arises naturally from the accelerated life testing problem and uses linear Bayesian methods for inference. The advantage of the procedure is that it does not require large number of items to be tested and that it can deal with both censored and uncensored data. Furthermore, the approach produces closed form inference results. We illustrate the use of our approach with some actual accelerated life test data.

1. INTRODUCTION AND OVERVIEW

In reliability studies it is a common practice to subject items to an environment which is more severe than the normal operating enviroment so that failures can be induced in a short amount of test time. A more severe environment can be created by increasing one or more of the stress levels, which constitute the environment, to values which are greater than their usual levels. Such tests are called *accelerated life* tests. The main problem with inference from accelerated life tests is that uncertainty statements about the failure behavior of the items at usual stress conditions have to be made using life length data from the more severe stress conditions. Most of the current literature on the accelerated life testing problem is based on the sample theoretic paradigm [see Meeker and Hahn (1985) for an up—to—date review] ; exceptions to these are Meinhold and Singpurwalla (1984) and Blackwell and Singpurwalla (1986) which involve a *Kalman Filter* formulation of the accelerated life testing problem.

In this paper we present a new procedure for inference from accelerated life tests. Our procedure is Bayesian and is based on a *dynamic general linear model* (DGLM) setup [see West, Harrison and Migon (1985), henceforth WHM] which arises naturally from the accelerated life testing scenario. Our procedure uses the *linear Bayesian approach* of WHM and produces closed form solutions for inference. The main advantage of the procedure is that it does not require large number of items to be tested at each stress level and that it can deal with both censored and uncensored data. Besides, the recursive nature of the results produced by the procedure facilitates its use on Personal Computers.

In our setup we assume that lifetimes obtained under all stress levels have an exponential distribution and make use of a particular time transformation function. The extension of our approach to other failure distributions such as the Weibull distribution is being investigated. The extension of the procedure to other time transformation functions is straight forward.

A synopsis of our paper is as follows:

In Section 2 we present the notation and preliminaries for the accelerated life testing problem. In Section 3 we introduce the *power law* as a time transformation function and present the DGLM setup for the problem. We also discuss the adoption of our procedure to other time transformation functions. In Section 4 we describe the inference procedure for the DGLM setup. Finally in Section 5, we illustrate the use of our approach by applying it to some actual accelerated life testing data published by Nelson (1972).

2. NOTATION AND PRELIMINARIES

Assume that testing is done in stages using k accelerated stress levels of decreasing intensity which are specified in advance and may or may not be equally spaced. During the i-th stage of testing, items are tested under an accelerated stress level denoted by S_i. Using the notation $S_i < S_j$, to denote that S_j is a more severe testing environment than S_i, we note that

$$S_1 > S_2 > \cdots > S_k > S_{k+1} , \tag{2.1}$$

where S_{k+1} denotes the use stress at which no testing is done.

At each stress level S_i, n_i items are tested for a predetermined and fixed time length τ_i. Let Y_{ij} denote the time to failure of the j-th item tested under stress level S_i, and y_{ij} denote the realization of Y_{ij}, for $j = 1, 2, \ldots, r_i \leq n_i$ where r_i denotes the number of failures observed during the time interval $(0, \tau_i]$.

We assume that the failure rate function for items tested under stress S_i is constant and denoted by λ_i. Thus, given λ_i the failure distribution under stress S_i is described by the exponential density

$$p(y_i \mid \lambda_i) = \lambda_i e^{-\lambda_i y_i} . \tag{2.2}$$

Furthermore, given λ_i, failure times for items tested under stress S_i are judged to be independent.

Given the r_i's and y_{ij}'s, our goal is to make inferences about the failure behavior of an item operating at use stress (normal) conditions S_{k+1}. In so doing, it is most common to assume a functional relationship between the failure rate and the applied stress level. Such relationship is known as an *acceleration* or *time transformation* function. Commonly used models for describing such relationship are the *Arrhenius Law*, the *Eyring Law*, and the *Power law*; see for example Mann, Schafer and Singpurwalla (1974), p. 421. In what follows, we will focus attention on the popular Power Law and present the DGLM setup for the accelerated life testing problem. In general, the use of any of these laws should be based on the physics of failure for the problem at hand.

3. THE DGLM SETUP FOR ACCELERATED LIFE TESTING

Under the Power Law, we write

$$\lambda_t = \alpha_t S_t^{\beta_t} , \tag{3.1}$$

where α and β are unknown coefficients which describe the stress effect on failure rate. The subscripts associated with α and β imply the fact that the time transformation function might be changing from one stress level to another. As noted by Blackwell and Singpurwalla (1986) this is likely to happen due to the changes in the basic failure mechanism with changes in the stress level.

We can linearize (3.1) by taking natural logarithms on both sides and write

$$\eta_t = \log \alpha_t + \beta_t \log S_t , \tag{3.2}$$

where $\eta_t = \log \lambda_t$. We define $\underset{\sim}{\theta}_t' = (\log \alpha_t \quad \beta_t)$, $\underset{\sim}{F}_t' = (1 \quad \log S_t)$ and write (3.2) as

$$\eta_t = \underset{\sim}{F}_t' \, \underset{\sim}{\theta}_t . \tag{3.3}$$

We note that (3.3) provides us with a *guide relationship* in the sense of WHM and that $\underset{\sim}{\theta}_t$ is the underlying state vector.

Next we describe how the time transformation function is changing from one stress level to another by specifying the *system (evolution)*

equation of the model as

$$\underset{\sim}{\theta}_i = \underset{\sim}{\theta}_{i-1} + \underset{\sim}{w}_i\,, \tag{3.4}$$

where $\underset{\sim}{w}_i$ is a random innovation term. We note that in (3.4) the state vector $\underset{\sim}{\theta}_i$'s are assumed to be constant from one stress level to another, except for the random changes brought about by the innovations $\underset{\sim}{w}_i$. The distribution of $\underset{\sim}{w}_i$ is only partially specified through its first and second order moments. We use the notation,

$$\underset{\sim}{w}_i \sim [\,\underset{\sim}{0}\,,\,\underset{\sim}{W}_i\,]\,, \tag{3.5}$$

to denote the fact that $\underset{\sim}{w}_i$ has mean vector $\underset{\sim}{0}$ and variance-covariance matrix $\underset{\sim}{W}_i$. Typically, $\underset{\sim}{w}_i$ is independent of $\underset{\sim}{\theta}_{i-1}$.

Let D_i denote all the relevant information available at stage i after observing r_i and failure times $\{y_{ij}\}_{j=1}^{r_i}$ at stress level S_i; D_0 represents all relevant information available at stage 0 prior to any testing. At stage (i-1), we assume that the distribution of the state vector $\underset{\sim}{\theta}_{i-1}$ is partially described by the first and second order moments as

$$(\,\underset{\sim}{\theta}_{i-1}\,|\,D_{i-1}\,) \sim [\,\underset{\sim}{m}_{i-1}\,,\,\underset{\sim}{C}_{i-1}\,]\,. \tag{3.6}$$

Using the system equation and (3.6) we can write

$$(\,\underset{\sim}{\theta}_i\,|\,D_{i-1}\,) \sim [\,\underset{\sim}{m}_{i-1}\,,\,\underset{\sim}{R}_i\,]\,, \tag{3.7}$$

where $\underset{\sim}{R}_i = \underset{\sim}{C}_{i-1} + \underset{\sim}{W}_i$. Equation (3.7) provides us with a partial description of the prior distribution of the state vector $\underset{\sim}{\theta}_i$ before testing at stress level S_i. It is important to note that the innovation term $\underset{\sim}{w}_i$ provides an incŕease in uncertainty (represented by the addition of $\underset{\sim}{W}_i$) over the stress levels as $\underset{\sim}{\theta}_{i-1}$ changes to $\underset{\sim}{\theta}_i$. This loss of information about $\underset{\sim}{\theta}_i$ motivates the *discount concept* [see for example, Smith (1979) and Ameen and Harrison(1985)] used by WHM as a guide for the choice of $\underset{\sim}{W}_i$. The underlying idea is that the increase in uncertainty over the next stress level should be relative to that available at the present stress level, measured by $\underset{\sim}{C}_{i-1}$. The details associated with the choice of the discount factor are described by WHM. In our case, the discount factor can be chosen as function of (S_i/S_{i-1}) to take into account the relative magnitudes of the stresses.

From the guide relationship (3.3) we can write

$$f_i = E[\eta_i \mid D_{i-1}] = \underset{\sim}{F}_i' \underset{\sim}{m}_{i-1} ,$$
$$q_i = Var[\eta_i \mid D_{i-1}] = \underset{\sim}{F}_i' \underset{\sim}{R}_i \underset{\sim}{F}_i . \qquad (3.8)$$

We note that (3.8) provides us with only the first two moments of the prior distribution of η_i at stage (i-1). We can specify a full distributional form for the prior of η_i and use the relationships given by (3.8) to determine the parameters of the distribution. In order to obtain closed form solutions in the Bayesian analysis, we will specify a conjugate form for the prior of η_i

which is a loggamma density of the form

$$p(\eta_i \mid D_{i-1}) \propto \exp\{\ a_i\eta_i - \ b_i e^{\eta_i}\ \}, \tag{3.9}$$

where a_i and b_i are the prior parameters. We denote the density in (3.9) by $\mathcal{LG}(a_i\,,b_i)$. From (3.9) we may obtain

$$E[\eta_i \mid D_{i-1}] = \Psi(a_i) - \log(b_i),$$

and

$$Var[\eta_i \mid D_{i-1}] = \Psi'(a_i),$$

where $\Psi(\cdot)$ and $\Psi'(\cdot)$ are the *digamma* and *trigamma* functions respectively. We will specify the prior parameters a_i and b_i such that the first two moments of η_i agree with (3.8), that is,

$$\begin{aligned} \Psi(a_i) - \log(b_i) &= f_i, \\ \Psi'(a_i) &= q_i. \end{aligned} \tag{3.10}$$

In solving (3.10) for a_i and b_i some approximations to $\Psi(\cdot)$ and $\Psi'(\cdot)$ can be used [see for example, Cox and Lewis (1966), Ch. 2]. We note that specifying the prior distribution of η_i as $\mathcal{LG}(a_i\,,b_i)$ implies that the prior of λ_i at stage (i-1) is a gamma density with shape parameter a_i and scale parameter b_i. We will denote this density as $\mathcal{G}(a_i,b_i)$.

To summarize, the DGLM setup for the accelerated life testing problem is as follows:

$$p(y_t \mid \lambda_t) = \lambda_t e^{-\lambda_t y_t},$$

$$(\eta_t \mid D_{t-1}) \sim \mathcal{L}\mathcal{G}(a_t, b_t); \quad \eta_t = \log(\lambda_t), \tag{3.11}$$

$$(\underset{\sim}{\theta}_t \mid D_{t-1}) \sim [\, \underset{\sim}{m}_{t-1} \, , \underset{\sim}{R}_t \,],$$

where a_t and b_t are chosen according to the guide relationship $\eta_t = \underset{\sim}{F}_t' \, \underset{\sim}{\theta}_t$ so that $E[\eta_t \mid D_{t-1}] = f_t$ and $Var[\eta_t \mid D_{t-1}] = q_t$.

We note that the DGLM setup for the accelerated life testing problem can be easily modified for other time transformation functions. For example, under the Arrhenius Law

$$\lambda_t = \exp\{\alpha_t - \beta_t / S_t\} \, , \tag{3.12}$$

which implies that the guidance relationship $\eta_t = \log(\lambda_t)$ is given by equation (3.3) with $\underset{\sim}{\theta}' = (\alpha_t \ \ \beta_t)$ and $\underset{\sim}{F}_t' = (1 \ \ -1/S_t)$. Now the DGLM setup of (3.11) can be applied to the problem. Extension to the Eyring and other laws follows along the same lines.

4. INFERENCE RESULTS FOR DGLM SETUP

Given D_t, the available information after the i-th stage of testing, we need to update our inferences about η_t and $\underset{\sim}{\theta}_t$. The posterior distribution of η_t given D_t is obtained via the standard use of Bayes' theorem

$$p(\eta_t \mid D_t) \propto \mathcal{L}(\eta_t ; D_t) \; p(\eta_t \mid D_{t-1}),$$

where $\mathcal{L}(\eta_t;D_t)$ is the likelihood function for η_t given by

$$\mathcal{L}(\eta_t;D_t) = \exp\{\eta_t r_t - T_t e^{\eta_t}\}, \tag{4.1}$$

and $T_t = \sum_{j=1}^{r_t} y_{tj} + (n_t - r_t)\tau_t$ is the *total time on test* at stress level S_t. It can be shown that

$$(\eta_t | D_t) \sim \mathcal{LG}(a_t + r_t, b_t + T_t), \tag{4.2}$$

implying that $(\lambda_t | D_t) \sim \mathcal{G}(a_t + r_t, b_t + T_t)$. The posterior mean and variance of η_t are given by

$$\begin{aligned} g_t &= E[\eta_t | D_t] = \Psi(a_t + r_t) - \log(b_t + T_t), \\ p_t &= Var[\eta_t | D_t] = \Psi'(a_t + r_t). \end{aligned} \tag{4.3}$$

The next step is to obtain the posterior mean and variance of the state vector $\underset{\sim}{\theta}_t$. We note that the full form of the posterior distribution for $(\underset{\sim}{\theta}_t | D_t)$ is not available since the prior of $(\underset{\sim}{\theta}_t | D_{t-1})$ is only partially specified. Recognizing the fact that, given η_t, the observation model (2.2) does not depend on $\underset{\sim}{\theta}_t$, WHM developed a method for updating the first two moments of $(\underset{\sim}{\theta}_t | D_t)$ using the *linear Bayesian approach* of Hartigan (1969). The details of the method is omitted here and only the main result is presented.

The method described by WHM leads to the updating of the state vector as

$$(\underset{\sim}{\theta}_t \mid D_t) \sim [\underset{\sim}{m}_t\, ,\, \underset{\sim}{C}_t], \tag{4.4}$$

where

$$\begin{aligned} \underset{\sim}{m}_t &= \underset{\sim}{m}_{t-1} + \underset{\sim}{s}_t(g_t - f_t)/q_t, \\ \underset{\sim}{C}_t &= \underset{\sim}{R}_t - \underset{\sim}{s}_t \underset{\sim}{s}_t' \left\{ \frac{1-p_t/q_t}{q_t} \right\}, \end{aligned} \tag{4.5}$$

with $\underset{\sim}{s}_t = \underset{\sim}{R}_t \underset{\sim}{F}_t$. We note that the updating equations for $\underset{\sim}{\theta}_t$ in the DGLM setup have the same form as the well-known Kalman filter recursions [see for example Meinhold and Singpurwalla (1983)].

After performing the test at the last stress level S_k, our aim is to make inference about the life length of an item at use stress conditions S_{k+1}. In other words, given D_k we wish to make uncertainty statements about Y_{k+1} and λ_{k+1} (or equivalently about η_{k+1}). Given D_k we have

$$(\eta_k \mid D_k) \sim \mathcal{L}\mathcal{G}(a_k + r_k\, , b_k + T_k),$$

and

$$(\underset{\sim}{\theta}_k \mid D_k) \sim [\, \underset{\sim}{m}_k\, ,\, \underset{\sim}{C}_k\,].$$

We describe our uncertainty about the state vector $\underset{\sim}{\theta}_{k+1}$ using the system equation; that is,

$$(\underset{\sim}{\theta}_{k+1} \mid D_k) \sim [\, \underset{\sim}{m}_k\, ,\, \underset{\sim}{R}_{k+1}\,], \tag{4.6}$$

where $\underset{\sim}{R}_{k+1} = \underset{\sim}{C}_k + \underset{\sim}{W}_{k+1}$. From the guide relationship of (3.3), we obtain

$f_{k+1} = \underset{\sim}{F}'_{k+1} \underset{\sim}{m}_k$ and $q_{k+1} = \underset{\sim}{F}'_{k+1} \underset{\sim}{R}_{k+1} \underset{\sim}{F}_{k+1}$. Using the DGLM setup of (3.11), we describe uncertainty about η_{k+1} via the density

$$(\eta_{k+1} \mid D_k) \sim \mathcal{L}\mathcal{G}[a_{k+1} , b_{k+1}], \tag{4.7}$$

where a_{k+1} and b_{k+1} are chosen such that $\Psi(a_{k+1}) - \log(b_{k+1}) = f_{k+1}$ and $\Psi'(a_{k+1}) = q_{k+1}$. Thus, inference about the use stress failure rate is made by using the probability density $(\lambda_{k+1} \mid D_k) \sim \mathcal{G}[a_{k+1}, b_{k+1}]$.

Inference about the life length of an item operating at use stress conditions is obtained using the predictive density

$$\begin{aligned} p(y_{k+1} \mid D_k) &= \int_0^\infty p(y_{k+1} \mid \lambda_{k+1}) \, p(\lambda_{k+1} \mid D_k) \; d\lambda_{k+1} \\ &= \frac{a_{k+1}(b_{k+1})^{a_{k+1}}}{(b_{k+1} + y_{k+1})^{a_{k+1}+1}} , \end{aligned} \tag{4.8}$$

which is a Pareto density with reliability function

$$R(y_{k+1} \mid D_k) = \frac{(b_{k+1})^{a_{k+1}}}{(b_{k+1} + y_{k+1})^{a_{k+1}}} . \tag{4.9}$$

A nice feature of the DGLM setup is that, at any stress level S_i, we can make inference about the failure behavior at any stress $S_j < S_i$ (that is, $i < j$) and at the use stress. Assume that we have tested items at stress level S_i and we have $(\ell\text{-}1)$ stages of testing ahead, that is, $\ell = k-i+1$. We

define

$$\hat{\underset{\sim}{m}}_t(\ell) = E[\,\underset{\sim}{\theta}_{t+\ell}\,|\,D_t],$$

and

$$\hat{\underset{\sim}{C}}_t(\ell) = \mathrm{Var}[\,\underset{\sim}{\theta}_{t+\ell}\,|\,D_t],$$

where $\hat{\underset{\sim}{m}}_t(0) = \underset{\sim}{m}_t$ and $\hat{\underset{\sim}{C}}_t(0) = \underset{\sim}{C}_t$. Using the system equation of the DGLM setup we obtain

$$\begin{aligned} \hat{\underset{\sim}{m}}_t(\ell) &= \underset{\sim}{m}_t\,, && \text{for } \ell \geq 0\,, \\ \hat{\underset{\sim}{C}}_t(\ell) &= \hat{\underset{\sim}{C}}_t(\ell\text{-}1) + \underset{\sim}{W}_{t+\ell} && \text{for } \ell > 0. \end{aligned} \tag{4.10}$$

Similarly, using the guide relationship we write

$$\begin{aligned} \hat{f}_t(\ell) &= E[\,\eta_{t+\ell}\,|\,D_t\,] = \underset{\sim}{F}'_{t+\ell}\,\hat{\underset{\sim}{m}}_t(\ell), \\ \hat{q}_t(\ell) &= \mathrm{Var}[\,\eta_{t+\ell}\,|\,D_t\,] = \underset{\sim}{F}'_{t+\ell}\,\hat{\underset{\sim}{C}}_t(\ell)\,\underset{\sim}{F}_{t+\ell}. \end{aligned} \tag{4. 11}$$

Thus, we can use (4.10) and (4.11) in the DGLM setup and make inference about the failure rate at stress levels $S_j < S_t$ given D_t.

5. EXAMPLE

The method was applied to some accelerated life test data taken from Nelson (1972). This data, given in Table 5.1, represents the times to breakdown of an insulating fluid with subjected to various voltage levels. The insulating fluid is tested at accelerated stress levels 26, 28, 30, 32, 34,

36, and 38 Kv, and inference is to be made concerning the breakdown times for the the insulating fluid at 20 Kv (use stress). In our approach we assume that testing was done in a sequential manner from highest to lowest stress level.

TABLE 5.1

Times to Breakdown of an Insulating Fluid (in Minutes)
Under Various Values of the Stress

38Kv	36Kv	34Kv	32Kv	30Kv	28Kv	26Kv
.09	.35	.19	.27	7.74	68.85	5.79
.39	.59	.78	.40	17.05	108.29	1579.52
.47	.96	.96	.69	20.46	110.59	2323.70
.73	.99	1.31	.79	21.02	426.07	
.74	1.69	2.78	2.75	22.66	1067.60	
1.13	1.97	3.16	3.91	43.40		
1.40	2.07	4.15	9.88	47.30		
2.38	2.59	4.67	13.95	139.07		
	2.71	4.85	15.93	141.12		
	2.90	6.50	27.80	175.88		
	3.67	7.35	53.24	194.90		
	3.99	8.01	82.85			
	5.35	8.27	89.29			
	13.77	12.06	100.58			
	25.50	31.75	215.10			
		32.52				
		33.91				
		36.71				
		72.89				

Due to our unfamiliarity with the problem at hand, the selection of the prior parameter values is made for illustrative purposes only. We use the Power Law as our time transformation function and specify our prior parameter values as

$$\underset{\sim}{m}'_0 = \begin{bmatrix} -15.0 & 10.0 \end{bmatrix}, \qquad \underset{\sim}{C}_0 = \begin{bmatrix} 0.5 & 0.0 \\ 0.0 & 0.5 \end{bmatrix}$$

and

$$\underline{W}_i = \begin{bmatrix} 0.01 & 0.0 \\ 0.0 & 0.01 \end{bmatrix}$$

for all stages i.

Plots of the posterior means of $\text{Log}\,\alpha_i$ and β_i, (which are the components of $\underline{m}_i$) for each stage of testing are presented in Figures 5.1 and 5.2 respectively. Note that there is an overall downward trend in both graphs and that after a strong decline from the prior mean values, a more stable behavior is exhibited. Thus

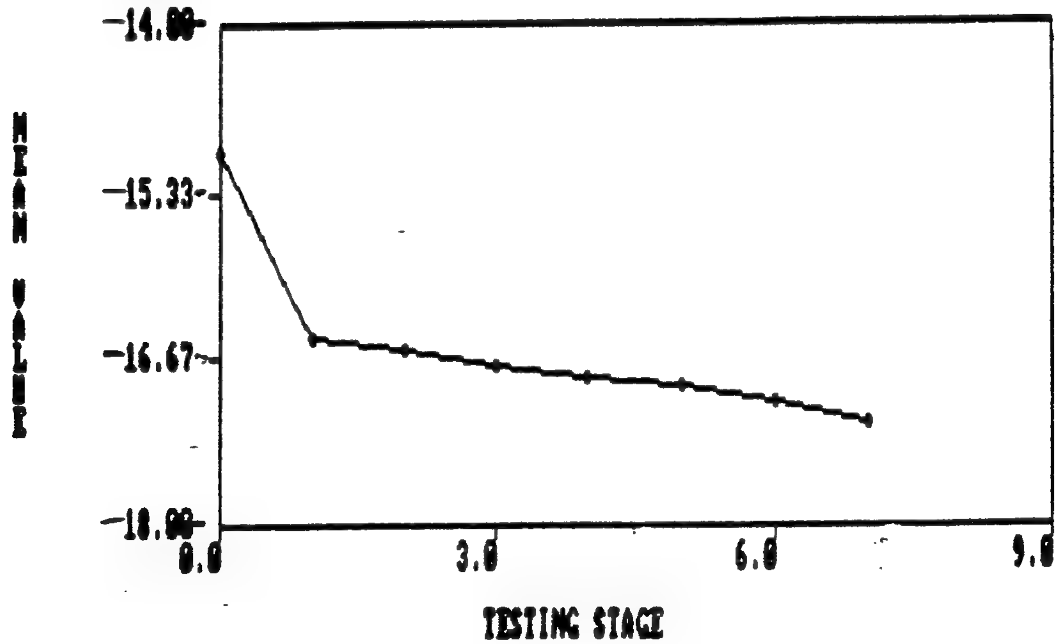

Fig. 5.1 Posterior Means of $\log \alpha_i$

implying that our prior belief about the effect of stress on failure behavior of the units was stronger than that exhibited by the data.

In Table 5.2 we present the $E[\lambda_j \mid D_i]$, $j = i, i+1, \ldots, k, k+1$ for each testing stage i. Columns of this table exhibit our assessment of the failure behavior of the system (insulating fluid) under the different levels of stress based on the total

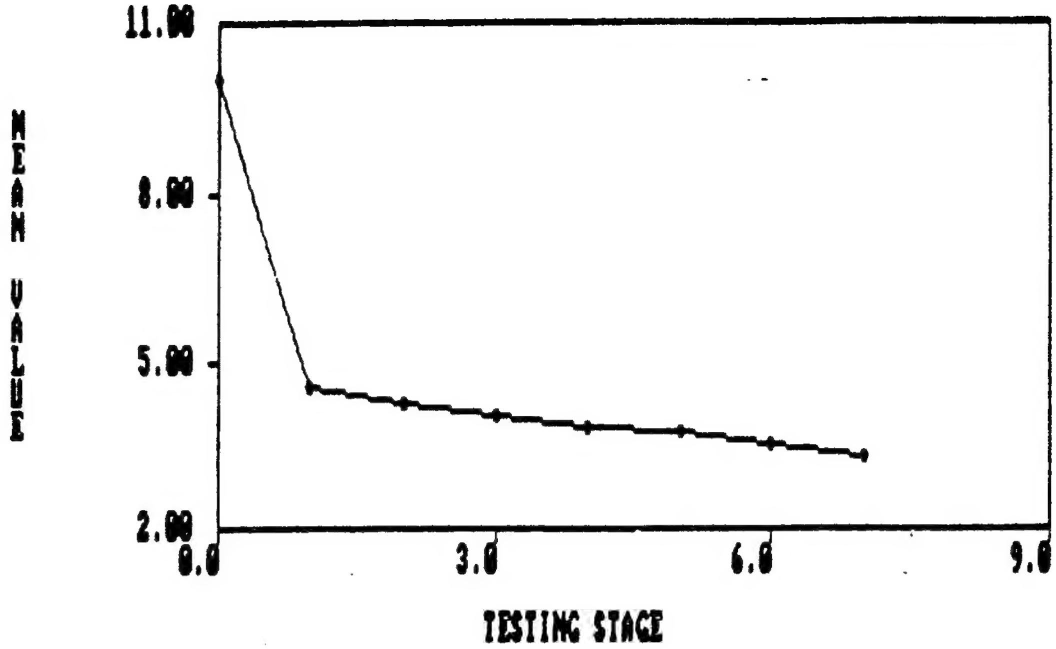

Fig. 5.2 Posterior Means of β_t

amount of information available after each stage of testing. In Figure 5.3 we plot the last row of this table to illustrate the evolution of the failure rate predictions for use stress. In addition, in Figure 5.4 we illustrate the predictive reliability functions for use stress after testing stages 1, 3, 5,and 7.

TABLE 5.2

Failure Rate Predictions at Different Testing Stages

PREDICTION FOR STAGE	TESTING STAGE 1	2	3	4	5	6	7
1	1.1102						
2	0.8155	0.2546					
3	0.6288	0.1944	0.0810				
4	0.4773	0.1503	0.0622	0.0293			
5	0.3559	0.1142	0.0480	0.0234	0.0154		
6	0.2601	0.0852	0.0363	0.0171	0.0115	0.0047	
7	0.1857	0.0622	0.0270	0.0129	0.0087	0.0034	0.0015
8 (use stress)	0.0563	0.0204	0.0094	0.0047	0.0033	0.0014	0.0006

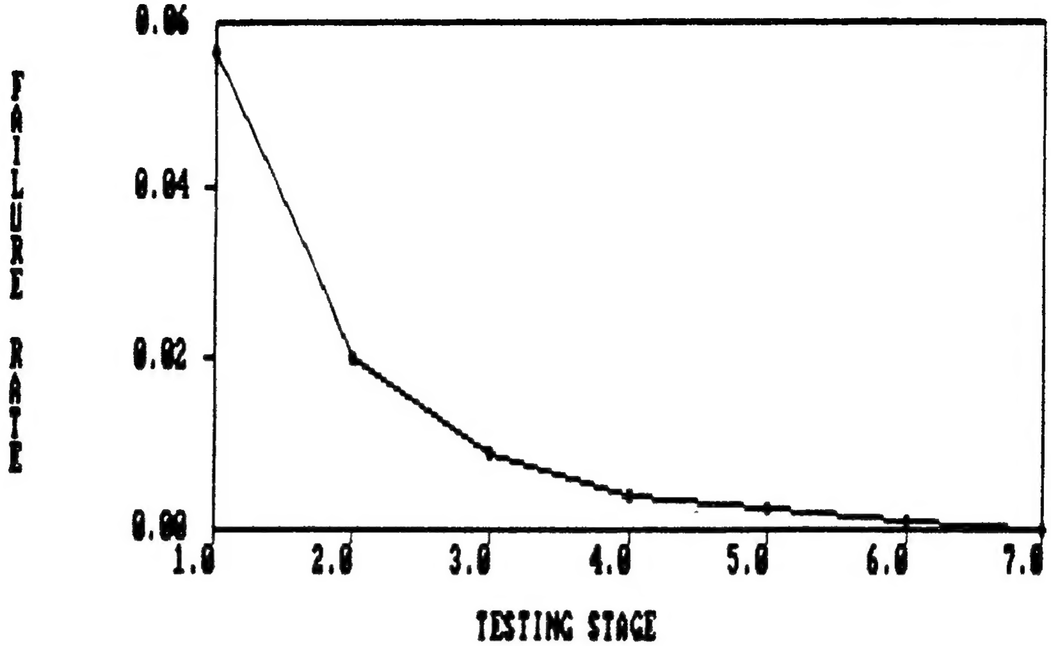

Fig. 5.3 Failure Rate Predictions for Use Stress at Different Testing Stages

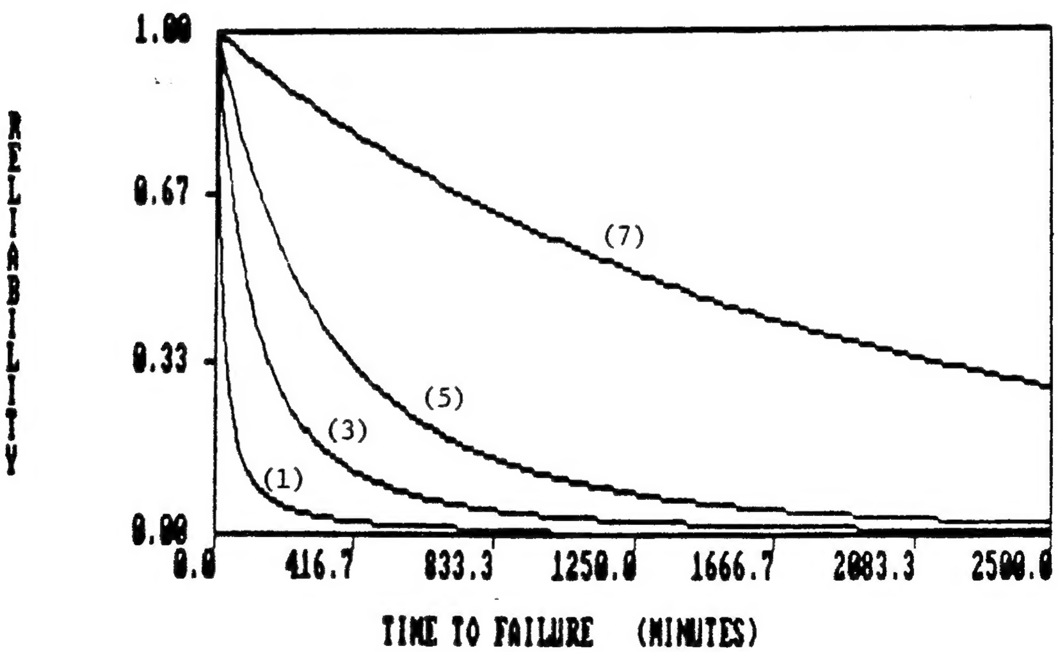

Fig. 5.4 Predicitive Reliability Function for Use Stress at Testing Stages i = 1, 3, 5, and 7

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REFERENCES

[1] J. R. M. Ameen, P. J. Harrison, "Normal discount Bayesian Models", in *Bayesian Statistics* 2, eds. J. M. Bernardo, M. H. DeGroot, D. V. Lindley, A. F. M. Smith, North–Holland, 1985, pp.271-298 .

[2] L. M. Blackwell, N. D Singpurwalla, "Inference from accelerated life tests using filtering in colored noise", Technical Report GWU/IRRA/TR-86/11, Institute for Reliability and Risk Analysis, George Washington University, 1986.

[3] D. R. Cox, P. A. W. Lewis, *Statistical Analysis of Series of Events*, Methuen, 1966.

[4] J. A. Hartigan, "Linear Bayesian Methods", *Journal of the Royal Statistical Society, Series B*, vol 31, 1969, pp 446-454.

[5] N. R. Mann, R. E. Schafer, N. D. Singpurwalla, *Methods for Statistical Analysis of Reliability and Life Data*, John Wiley and Sons, 1974.

[6] W. Q. Meeker, G. J. Hahn, "How to plan an accelerated life test – some practical guidelines", *American Society for Quality Control*, vol. 10, 1985.

[7] R. E. Meinhold, N. D. Singpurwalla, "Understanding the Kalman filter", applications", *American Statistician*, vol 37, 1983, pp. 123-127.

[8] R. E. Meinhold, N. D. Singpurwalla, "A Kalman filter approach to accelerated life testing – a preliminary development", in *Stochastic Failure Models, Replacement and Maintenance Policies, and Accelerated Life Testing*, ed. M. Abdel-Haneed, E. Cinlar, J. Quinn, Academic Press, 1984, pp. 169-175.

[9] W. B. Nelson, "Graphical analysis of accelerated life test data with the inverse power law model", *IEEE Trans. Reliability*, R-21, 1972, pp.2-11.

[10] J. Q. Smith, "A generalization of the Bayesian steady forecasting model", *Journal of the Royal Statistical Society, Series B*, vol 41, 1979, pp 378-387.

[11] M. West, P. J. Harrison, H. S. Migon, "Dynamic generalized linear models and Bayesian forecasting", *Journal of the American Statistical Association*, vol 80, 1985, pp 73-97.